Lecture 11/16/23 : Power Functions

Defn: A function of the form $f(x) = Kx^p$ where $K \neq 0$ and p is any constant is called a power function.

Ex: Which of the following are power functions? If so write in the form Kx^p

1) $g(t) = \frac{10}{2\sqrt[3]{t}}$ Yes $\quad \frac{10}{8} t^{-1/2}$

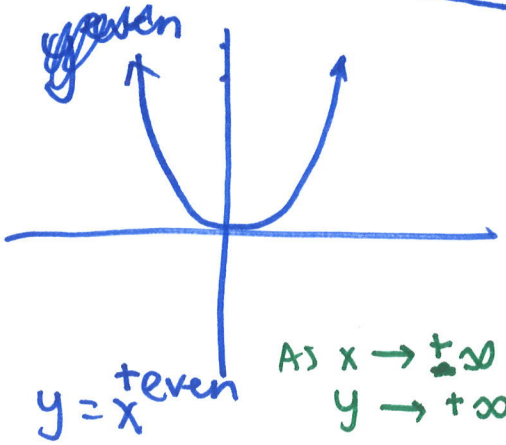
2) $C(t) = 12(1.1^x)^3$ No!

3) $P(x) = \ln(4^x) = x \ln(4)$ Yes!

4) $K(t) = 2.25t^2 + 4t^2 = 6.25t^2$ Yes!

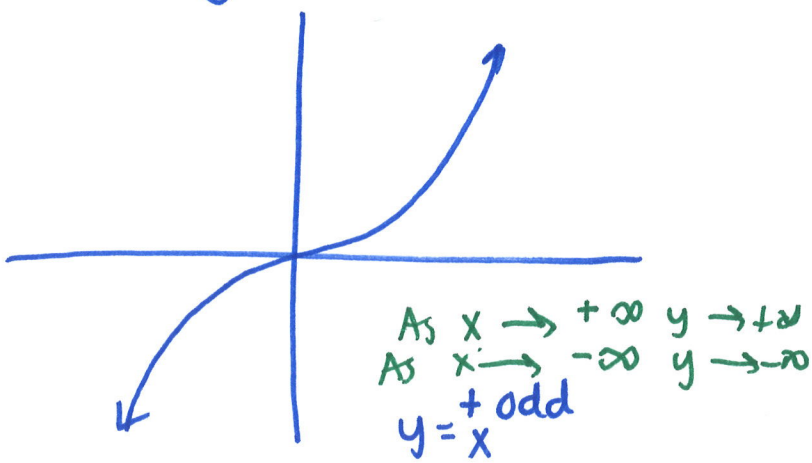
5) $x^3 + x^2$ No!

Graphs of Power Functions + Long Run Behavior



$y = x^{+\text{even}}$

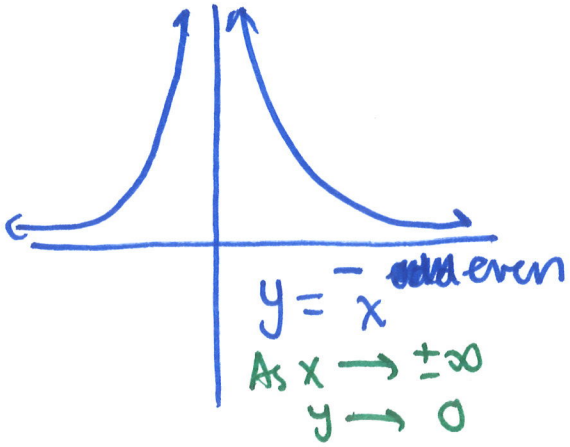
As $x \to \pm\infty$, $y \to +\infty$



As $x \to +\infty$, $y \to +\infty$

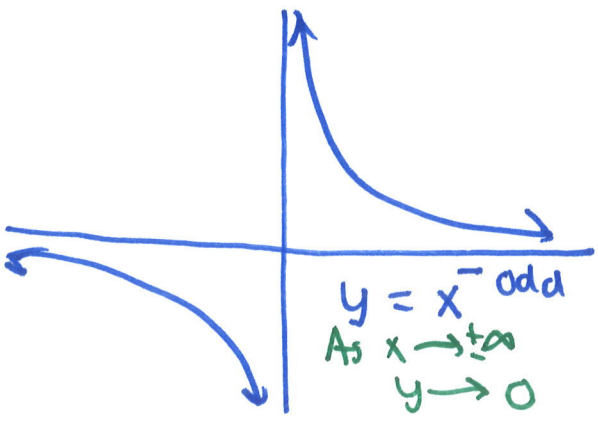
As $x \to -\infty$, $y \to -\infty$

$y = x^{+\text{odd}}$



$y = x^{-\text{even}}$

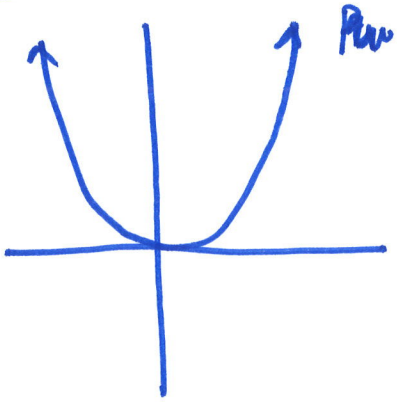
As $x \to \pm\infty$, $y \to 0$



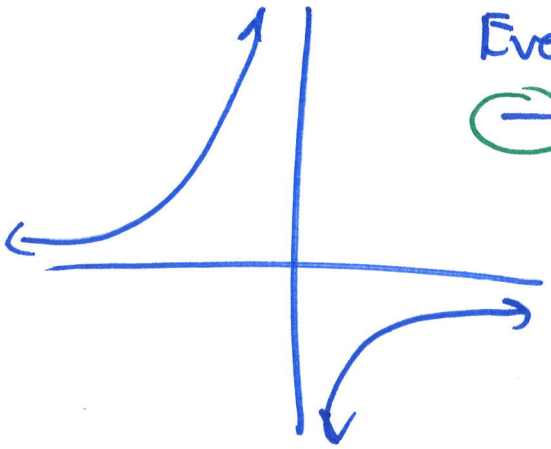
$y = x^{-\text{odd}}$

As $x \to \pm\infty$, $y \to 0$

Ex: Even Odd



Even (Even) Odd
− (+)



Even (Odd)
(−) +

Ex: As $t \to \infty$, $4t^4 \to \infty$

As $x \to -\infty$, $3x^{-3} \to 0$

As $z \to \infty$, $2z^{-6} + 17 \to 17$

As $y \to \infty$, $3y^7 - 1 \to \infty$

#7 a) A power function is of the form

$$f(x) = Kx^P$$

We have $(2, 4)$ and $(1/3, 1/54)$ lying on $f(x)$ so

$$4 = f(2) = K2^P \quad (1)$$

$$1/54 = f(1/3) = K(1/3)^P \quad (2)$$

Divide $\frac{(1)}{(2)} = \frac{K2^P}{K(1/3)^P} = \frac{2^P}{(1/3)^P} = \left(\frac{2}{1/3}\right)^P = \frac{4}{1/54}$ ✓

So, $6^P = 216$. Hence $P = \frac{\ln(216)}{\ln(6)}$ (Logs!)

Now, $4 = K2^P = K2^{\frac{\ln(216)}{\ln(6)}} \Rightarrow K = 4/2^{\frac{\ln(216)}{\ln(6)}}$

$$f(x) = \frac{4}{2^{\ln(216)/\ln(6)}} \cdot x^{\ln(216)/\ln(6)}$$